PHYSICS

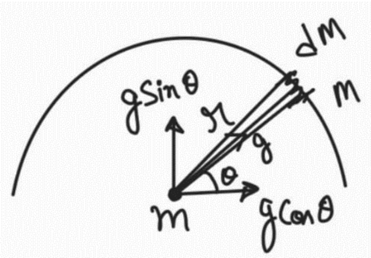
1. A metallic wire of uniform mass density having mass M and length l is bent to form a semicircle. A point mass m is kept at the centre of the semicircle. Find the gravitational forced experienced by m.

Ans. $\frac{2\pi GMm}{L^2}$

Sol. $r = \frac{L}{\pi}$

$dg = \frac{Gdm}{r^2}\sin\theta$

$= \frac{G}{r^2}\frac{M}{L}rd\theta\sin\theta$



$= \frac{G}{g} = \frac{G}{r}.\frac{M}{L}\int_0^{\pi}\sin\theta d\theta$

$g = \frac{GM}{rL}(2)$

$F = mg$

$= m^2\frac{GM}{rL}$

$= \frac{2GMm}{L}\frac{\pi}{L}$

$= \frac{2\pi GMm}{L^2}$

2. 5 convex lens are kept together each having power of 25 D. Find the focal length.

Ans. 0.8 cm

Sol. $P_{eq} = P \times 5$

$= 25 \times 5$

$= 125D$

$\frac{1}{f_{eq}} = 125\,m$

$= \frac{100}{125}$ cm

$= \frac{4}{5}$ cm

= 0.8 cm

3. Position of a particle is related to time as given equation

$x = t^4 + 6t^2 + 2t$

Find its acceleration at t = 5 sec.

Ans. 480 m/s^2

Sol. $V = \frac{dx}{dt}$

$V = 4t^3 + 18t^2 + 2$

$a = \frac{dV}{dt}$

$= 12t^2 + 36t$

At t = 5 sec

$a = 12 \times 25 + 36 \times 5$

$= 300 + 180$

$= 480\ m/s^2$

4. A body moving with constant acceleration covers 102.5 m in n^{th} second of its motion and covers 115.0 m in $(n + 2)^{th}$ second then find its acceleration.

Ans. 6.25 m/s^2

Sol. Let, acceleration = a (constant)

$$S_{n^{th}} = u + \frac{a}{2}[2n - 1] = 102.5 \qquad \text{...(i)}$$

$$S_{(n+2)^{th}} = u + \frac{a}{2}[2(n+2) - 1] = 115$$

$$\Rightarrow u + \frac{a}{2}[2n+3] = 115 \qquad \text{...(ii)}$$

by using (i) and (ii)

$$102.5 - \frac{a}{2}[2n-1] + \frac{a}{2}[2n+3] = 115$$

$$\Rightarrow 102.5 + \frac{a}{2} + \frac{3a}{2} = 115$$

$$\Rightarrow 2a = 115 - 102.5$$

$$a = \frac{12.5}{2} = 6.25 m/s^2$$

5. A particles of mass m dropped from height h above the ground. After collision, rises to height h/2, Then loss in energy during collision and speed of particle just before collision respectively are.

(1) 50%, $\sqrt{2gh}$ (2) 40%, $\sqrt{2gh}$ (3) 50%, $\sqrt{gh}$ (4) 40%, $\sqrt{gh}$

Ans. (1)

Sol. $\Delta E = mg\frac{h}{2} - mgh = -mg\frac{h}{2}$

i.e. 50% loss in energy

$v = \sqrt{2gh}$

6. If the electric field vector at a point in an electromagnetic wave is given by $\vec{E} = 40\cos\omega\left(t - \frac{z}{c}\right)\hat{i}$ then corresponding $\vec{B}$ will be:

Sol. $\vec{E} = 40\cos\omega\left(t - \frac{z}{c}\right)\hat{i}$

$$|\vec{E}| = 40\cos\omega\left(t - \frac{z}{c}\right)$$

$$\frac{|\vec{E}|}{|\vec{B}|} = C$$

$$|\vec{B}| = \frac{40}{C}\cos\omega\left(t - \frac{z}{C}\right); \text{ also } \vec{E}.\vec{B} = 0$$

7. Infinite charge sheet in xy plane of surface charge density σ and infinite long wire of linear charge density λ placed at (0, 0, 4) and $\sigma = 2\lambda$. Then net electric field (0, 0, 2).

Ans. $E_{net} \Rightarrow \frac{\lambda}{\varepsilon_0}\left[\frac{2\pi r - 1}{2\pi r}\right]$ N/C

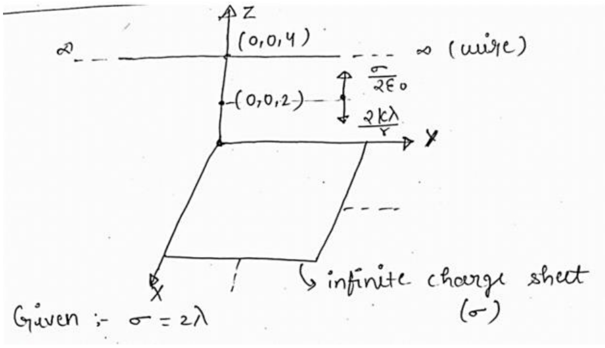
Sol. Given : $\sigma = 2\lambda$

$$E_{net} = \frac{\sigma}{2\varepsilon_0} - \frac{2K\lambda}{r}$$

$$E_{net} = \frac{2\lambda}{2\varepsilon_0} - \frac{2\lambda}{4\pi\varepsilon_0 r}$$

$$E_{net} = = \frac{2\lambda}{2\varepsilon_0} - \frac{2\lambda}{4\pi\varepsilon_0 r}$$

$$\Rightarrow \frac{\lambda}{\varepsilon_0}\left[\frac{2\pi r - 1}{2\pi r}\right] \text{N/C}$$



8. A hollow cylinder and solid sphere of same mass and radius are rolling with same initial velocity v on a rough inclined plane. Find the ratios of their kinetic energies and maximum height reached by them.

Ans. $\frac{10}{7}$

Sol. $K_{cylinder} = \frac{1}{2}MV^2 + \frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}MV^2 + \frac{1}{2}(MR^2)\left(\frac{V}{R}\right)^2$

$= MV^2$

$K_{sphere} = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}MV^2$

$= \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{V}{R}\right)^2 + \frac{1}{2}MV^2$

$= \frac{1}{5}MV^2 + \frac{1}{2}MV^2$

$= \frac{7}{10}MV^2$

$\Rightarrow \frac{K_{cylinder}}{k_{sphere}} = \frac{10}{7}$

At top point kinetic energy will convert into potential energy

$\frac{Mgh_{cylinder}}{Mgh_{sphere}} = \frac{10}{7}$

$\Rightarrow \frac{h_{cylinder}}{h_{sphere}} = \frac{10}{7}$

9. In given equation $y = 2A\sin\left(\frac{2\pi nt}{\lambda}\right)\cos\left(\frac{2\pi x}{\lambda}\right)$. Find the dimension of n.

Ans. $[n] = [L^1T^{-1}]$

Sol. $[n] \Rightarrow \frac{[2\pi nt]}{[\lambda]} + M^0L^0T^0$

$\frac{[n][T^1]}{[L^1]} = M^0L^0T^0$

$[n] = [L^1T^{-1}]$

10. When a conducting platinum wire is placed in ice, its resistance is 8Ω and when placed in steam it is 10Ω. Find the resistance of wire at 400°C.

Ans. 8.8Ω

Sol. $R_T = R_0(1 + \alpha\Delta T)$

R_0 at 0° $\Rightarrow 8\Omega$

R_T at 100°C $\rightarrow 10\Omega$

$10 = 8(1 + \alpha(100))$

$$\frac{10}{8} = 1 + 100\alpha$$

$$\left(\frac{10}{8} - 1\right) \times \frac{1}{100} = \alpha$$

$$\alpha = \frac{2}{8} \times \frac{1}{100}$$

$$\alpha = \frac{1}{400}$$

R at 40°

$R = R_0(1 + \alpha\Delta T)$

$$= 8\left(1 + \frac{1}{400} \times 40\right)$$

$$= 8\left(1 + \frac{1}{10}\right)$$

$$= \frac{11 \times 8}{10}$$

R = 8.8Ω

11. Fractional error in image distance and object distance are $\frac{\Delta v}{v}$ and $\frac{\Delta u}{u}$ then find the fractional error in focal length of the given spherical mirror.

Ans. $\Rightarrow \frac{df}{f} = \frac{uv}{u+v}\left[\frac{dv}{v^2} + \frac{du}{u^2}\right]$

Sol. $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$$\frac{1}{f} = \frac{u+v}{uv}$$

$$f = \frac{uv}{u+v}$$

$$\Rightarrow -\frac{1}{f^2}df = -\frac{dv}{v^2} - \frac{du}{u^2}$$

$$\Rightarrow \frac{df}{f} = f\left[\frac{1}{v}\frac{dv}{v} + \frac{1}{u}\frac{du}{u}\right]$$

$$\Rightarrow \frac{df}{f} = \frac{uv}{u+v}\left[\frac{dv}{v^2} + \frac{du}{u^2}\right]$$

12. Instantaneous current in a circuit is zero. In which of the options voltage will be maximum.

(a) L (b) C (c) R (d) LC

(1) ABD (2) B (3) BC (4) D

Ans. (1)

Sol. Phase difference between current and voltage is 90°.
So, possible circuit are (A), (B) and (D).

13. x and y coordinates of a body performing some motion is given as:

$x = 3 + 4t$

$y = 3t^2 + 4t$

Identify the trajectory of motion.

(1) Parabola (2) Circular (3) Straight line (4) Hyperbola

Ans. (1)

Sol. $x = 3 + 4t \Rightarrow t = \dfrac{x-3}{4}$ ....(1)

$y = 3t^2 + 4t$ ....(2)

equation (1) in (2)

$$y = 3\frac{(x-3)^2}{16} + 4\frac{(x-3)}{4}$$

$$\Rightarrow y = \frac{3}{16}(x^2 + 9 - 6x) + (x-3)$$

$$\Rightarrow y = \frac{1}{16}\left[3x^2 + 27 - 18x + 16x - 48\right]$$

$$y = \frac{1}{16}\left[3x^2 - 2x - 21\right]$$

$\Rightarrow$ it is quadratic in x

$\Rightarrow$ its trajectory is parabola.

14. Choose the correct graph for kinetic energy vs r for an electron revolving around a infinite line of charge.

Ans. Theoretical

Sol. Net force acting towards centre = $\dfrac{mv^2}{r}$

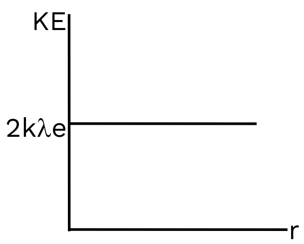
$F = q \times E$

$F = e \times 2k\dfrac{\lambda}{r}$

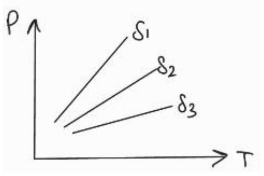
$$\Rightarrow \frac{mv^2}{r} = (e)\left(\frac{2k\lambda}{r}\right)$$

$mv^2 = 2k\lambda \times e$

$\Rightarrow KE = 2k\lambda e$



15. Pressure vs temperature graph is given for gas of different density. Compare ρ_1, ρ_2 and ρ_3?



Ans. $\rho_1 > \rho_2 > \rho_3$

Sol. $PM = \rho RT$

$\rho = \dfrac{PM}{RT}$

$\rho \propto \dfrac{P}{T}$

$\rho \propto$ slope

Hence $\rho_1 > \rho_2 > \rho_3$

16. Work done to expand the bubble of diameter 7 cm and surface tension 40 dyne/cm is 36960 erg. Find the radius of the expanded bubble?

Ans. 14 cm

Sol. Surface energy = T (area)

Bubble has tw surface of interface

$E_i = 2TS_i$

$E_f = 2TS_f$

$\Rightarrow$ Work done = $E_f - E_i$

$\Rightarrow 36960 = 2[TS_f - TS_i]$

$\Rightarrow 3690 = T\Delta S \times 2$

$\Rightarrow \Delta S = \dfrac{36960}{40 \times 2}$

$\Rightarrow \Delta S = 462 \text{ cm}^2$

$S_f - S_i = 462$

$\Rightarrow 4\pi r_f^2 = 462 + 4\pi r_i^2$

$$\Rightarrow r_f^2 = \frac{1}{4\pi}\left[462 + 4\pi \times \left(\frac{7}{2}\right)^2\right]$$

$$r_f^2 = \frac{1}{4\pi}[462 + 4\pi \times \frac{49}{4}]$$

$$= \frac{462 \times 7}{4 \times 22} + \frac{49}{4}$$

$$= r_f^2 = \frac{196}{4} = 49$$

$r_f = 7$ cm

diameter = 7 × 2 = 14 cm

17. De-Broglie wavelength of electron moving from n = 4 to n = 3 of a hydrogen is b(πa); Where a is bohr radius of the hydrogen atom. Find the value of b.

Ans. b = 2

Sol. $E = \dfrac{hc}{\lambda}$, $mvr = \dfrac{nh}{2\pi}$

$$\lambda = \frac{h}{mv} = \frac{2\pi r}{n}$$

$$(\lambda_1)_{n=4} = \frac{(2\pi)(a_0 n^2)}{n}$$

$$(\lambda_1)_{n=4} = (2\pi)(a_0 n) = 8\pi a_0$$

$$(\lambda_2)_{n=3} = 6\pi a_0$$

$\Delta\lambda = \lambda_1 - \lambda_2 = 8\pi a_0 - 6\pi a_0$

$\Delta\lambda = 2\pi a_0$

Therefore b = 2

18. An elastic string under tension of 3N has a length of 'a'. If length is 'b' then tension is 2N. Find tension when length is (3a – 2b).

Ans. $\dfrac{5F}{K}$

Sol. F = kx

$3F = Ka \Rightarrow a = \dfrac{3F}{K}$

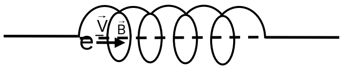
$2F = Kb \Rightarrow b = \dfrac{2F}{K}$

Now, $3a-2b = \frac{9F}{K} - \frac{4F}{K} = \frac{5F}{K}$

19. An electron projected inside the solenoid along its axis which carries constant current, then its trajectory would be:

Ans. Straight line

Sol.



$\vec{F} = q(\vec{V} \times \vec{B})$

$\vec{B}$ and $\vec{V}$ are parallel at axis of solenoid so, their cross product will be zero

i.e. $\vec{F} = 0$

So, electron will move with constant velocity in a straight line.

20. Current as a function of time is given as $i = 6 + \sqrt{56}\sin\left(100t + \frac{\pi}{3}\right)$ A. Find rms value of current.

Ans. 8 A

Sol. $i_{rms} = \sqrt{6^2 + \frac{(\sqrt{56})^2}{2}}$

$= \sqrt{36+28}$

$= \sqrt{64}$

= 8 A

21. In Celsius the temperature of a body increases by 40°C. The increasing temperature on Fahrenheit scale is:

Ans. 72°F

Sol. $T_F = \frac{9}{5}T_c + 32$

$\Delta T_f = \frac{9}{5}\Delta T_c$

$\Rightarrow \Delta T_F = \frac{9}{5} \times 40$

$\Rightarrow \Delta T_F = 72° F$

22. Force on a particle varies linearly with time(t) ($F \propto t$). Then select correct acceleration vs time graph.

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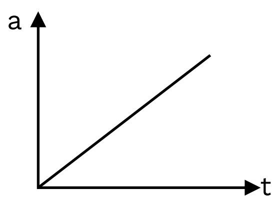

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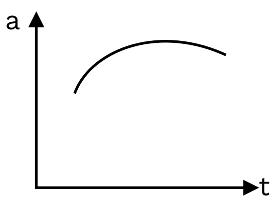

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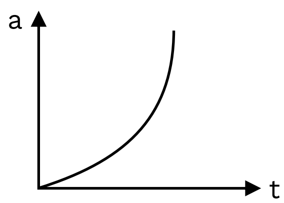

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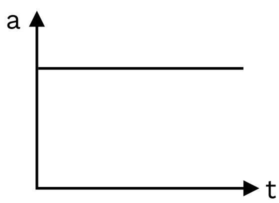


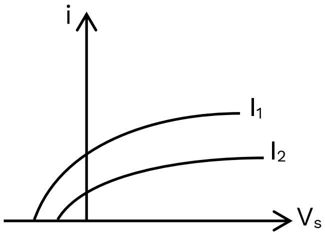
Ans. $\Rightarrow a \propto t$

Sol. $F = ma \Rightarrow a = \frac{F}{m}$

$\Rightarrow a \propto t$

23. Which graph correctly represents the photo current (i) vs stopping potential (V_s) for the same frequency but different intensity? (Here $I_1 > I_2$)

(1)



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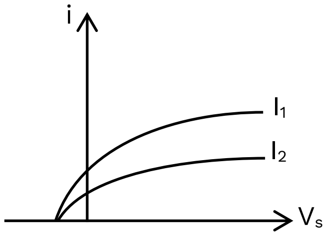

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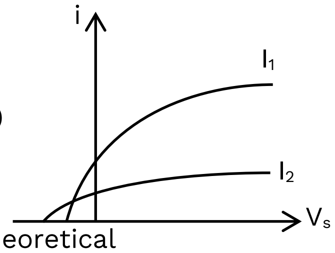

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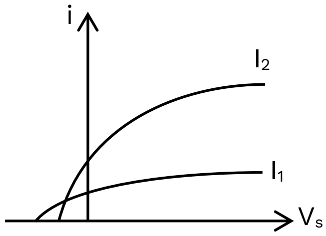
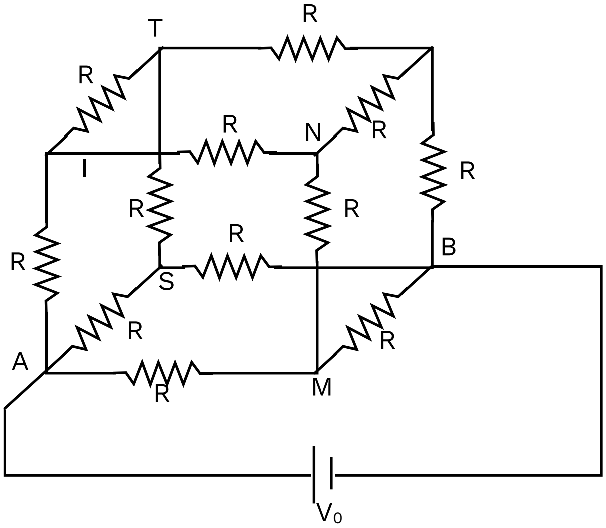


Ans. Theoretical

24. A cubical arrangement of 12 resistors each having resistance R is shown. Find I shown in the given circuit.



Ans. $\frac{V_0}{6R}$

Sol. $\frac{1}{R_{eq}} = \frac{1}{3R} + \frac{1}{R} = \frac{4}{3R}$

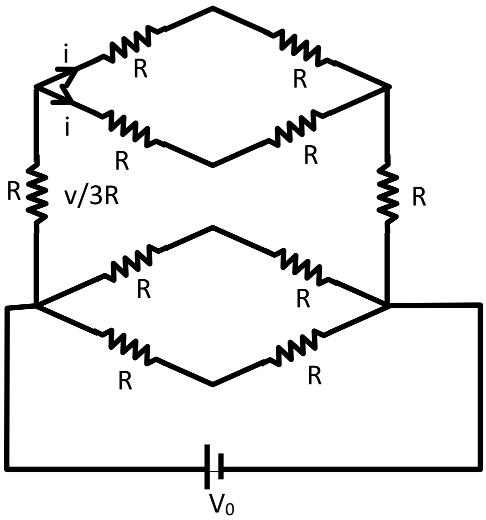
$R_{eq} = \frac{3R}{4}$

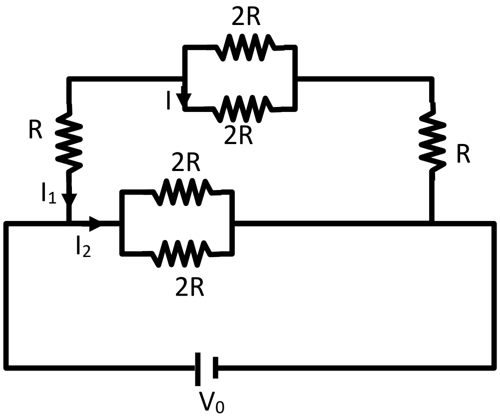
$\Rightarrow V_0 = IR_{eq}$

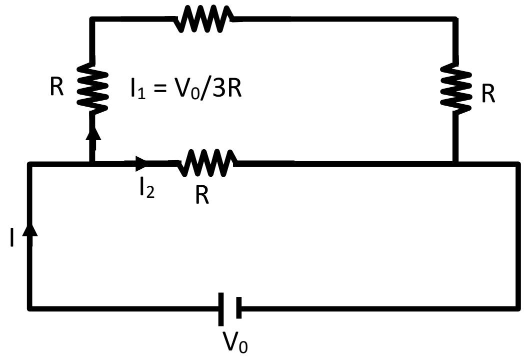
$\Rightarrow I = \frac{4V_0}{3R}$

So, $I_1 + I_2 = I$

$\Rightarrow$ in parallel combination, current is divided into inverse ratio of resistance







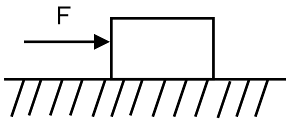
$\Rightarrow \frac{I_1}{I_2} = \frac{1}{3}$

$\Rightarrow I_1 + 3I_1 = I \Rightarrow I_1 = \frac{1}{4}I = \frac{V_0}{3R}$

Now, I_1 gets divided equally in both branches

$i = \frac{I_1}{2} = \frac{V_0}{3R} \times \frac{1}{2} \Rightarrow i = \frac{V_0}{6R}$

25. A wooden block is initially at rest on at rest a smooth surface. Now a horizontal force is applied on the block which increases linearly with time. The acceleration time (a-t) graph for the block would be:



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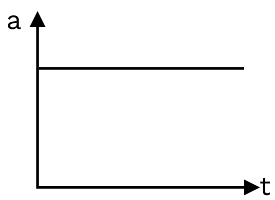

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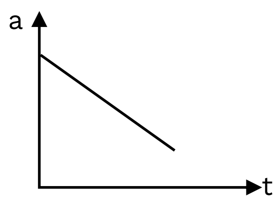

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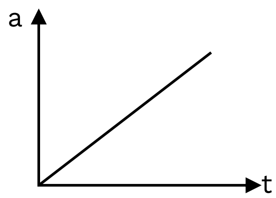

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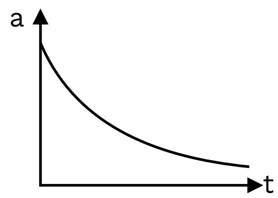
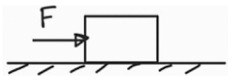


Ans. $F = \frac{k}{m}t$

Sol.


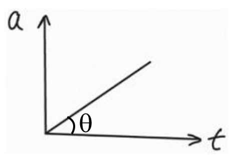
This horizontal force increases linearly with time

$F \propto t$

$F = kt + c$ $\quad (\therefore F = ma)$

$a = \frac{k}{m}t + \frac{c}{m}$

if, $\frac{c}{m} = 0$ $\quad \tan\theta = \frac{k}{m}$

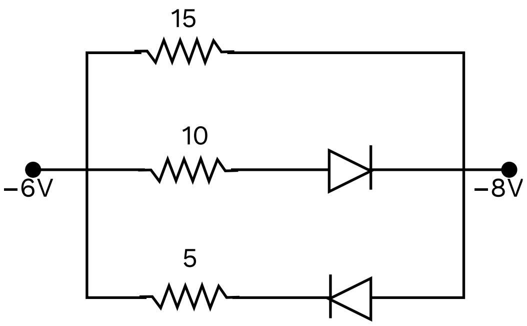


then :

$\Rightarrow F = kt$

$\Rightarrow a = F = \frac{k}{m}t$

26. Find R_{eq} ?



Sol. Below diode is in reverse bias so no current flow through it circuit looks like.

$R_{eq} = \frac{5}{2} = 2.5\Omega$